**Report – Assignment 3 – COT5405 – Analysis of Algorithms**

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**Question 1: Rod Cutting Problem**

**a. Psuedo code**:

**Description**:

In this problem there are recurring sub-problems which are computed and stored in memory for the first time and reused when the same sub-problem occurs. This helps us to avoid computing the same sub-problem multiple times. The dynamic programming approach used for this problem is called bottom-up approach in which we compute from base case till *n*.

1. Create memArray of size n+1 and set first element to 0.
2. Repeat the following for values from i = 1 till n:
   1. Set maxPrice to -infinity.
   2. For values from j = 0 to i-1 do the following:
      1. Calculate the maximum of max price and prices(j)+memArray(i-j-1)
   3. Set memArray(i) = maxPrice
3. The result will be in memArray(n).

**b. Time and Space Complexity:**

*Time Complexity:*

From 4th and 6th line of algorithm we get the following equation:

*Space Complexity:*

From 2nd line of algorithm, we have created an array of n + 1 elements and rest of them a few variables.

**Question 2: Dynamic-programming reflection**

The similarities between Dynamic Programming and the Divide and Conquer approach are that both concepts break the problem into sub-problems. Furthermore, all the decisions of the sub-problems are combined to answer the original problem.

Divide and Conquer is used when the sub problems are independent of each other and the result of one sub-problem cannot be re-used elsewhere. Dynamic Programming is used when there might be overlapping sub-problems, i.e., the result of each sub-problem is stored so that it can be re-used whenever necessary. Divide and Conquer follows a top-down approach whereas Dynamic Programming follows a bottom-up approach.

**Question 3: Shortest Path Counting**

**a. Psuedo code**:



**Description**:

We may presume that the rook starts out in the lower left corner of a chessboard with rows and columns numbered from 1 to 8 without losing generality. Let Q (i, j) be the number of shortest paths taken by the rook in the ith row and jth column from square (1,1) to square (i, j), where 1 <= i, j <= 8. Any such route would be made up of vertical and horizontal movements all aimed at the-same-target.  
For any 1 <= i, j <= 8, Q (i, 1) = Q (1, j) = 1 is self-evident. In general, the shortest path to square (i, j) is either through square (i, j -1) or through square (i-1, j).

Hence the recurrence relation is as follows:

1. Create an empty NxN matrix.
2. Initialize all cells in first row and first column with value 1.
3. Looping for all values of i from 2 till N:
   1. Looping for all values from j = 2 to N:
      1. matrix (i, j) = matrix (i, j-1) + matrix (i - 1, j)
4. Result will be in matrix (N, N).

**b. Time and Space Complexity:**

*Time Complexity:*

From 6th and 7th line of algorithm we get the following equation:

*Space Complexity:*

In line 2, an array of (N x N) elements are created. Therefore, the space complexity,

**c. Using elementary combinatorics:**

Any shortest path for a rook to move from one corner of a chessboard to the diagonally opposite corner can be thought of as 14 consecutive moves to adjacent squares, 7 of which being up while the other 7 being to the right. Hence, the total number of distinct shortest paths is equal to the number of different ways to choose 7 positions among the total of 14 possible positions,

which is equal to,

= 3432

Hence, by combinatorics, the number of shortest paths by which a rook can move from one corner of a chessboard to the diagonally opposite corner is 3432.

**Question 4: Implementing dynamic programming algorithms:**

**a. Psuedo code**

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**Description**:

In this bottom-up dynamic programming solution:

* First fill the first column in the matrix *memory*, of size is M x N, by inserting the value of ‘1’in the same location where a ‘0’ is found in *matrix;* reason being ‘0’ in matrix a 1x1 square and can be our possible answer. The variables *result, rowIndex, colIndex* are also updated simultaneously. The same step is repeated for the first row in *matrix.*
* For the subsequent rows, if a ‘0’ is found in *matrix*, the min(memory[i][j-1], memory[i-1][j], memory[i-1][j-1])+1 is calculated and inserted in memory[i][j].
* Otherwise, memory[i][j] is filled with a ‘0’ and subsequently, the result, rowIndex and colIndex variables are updated.
* At the end return the result, row, and column index variables. There is a separate function which prints the matrix using these returned values.

**b. Running time:**

In line 7 and 12 there are total M and N iterations, respectively. In line 18 and 19 there are M-1 \* N -1 iterations. Therefore, running time is as follows:

T (M, N) = M + N + (M – 1) x (N - 1)

≈ M + N + (MN – M – N + 1)

≈ MN - 1

≈

**c. Graph - Running time:**

Above graph is plotted between Size of Matrix from the provided list [(10, 10), (10, 100), (10, 1000), (100,1000) and (1000, 1000)] and Execution time. Solid plot in the graph is actual execution time taken by the program and dotted plot is O (n2) general plot. Therefore, this proves the running time of the algorithm.

**Graph – Memory Usage:**

Above graph is plotted between Size of Matrix from the provided list [(10, 10), (10, 100), (10, 1000), (100,1000) and (1000, 1000)] and Memory Usage.

Solid plot in the graph is actual memory used by the program and dotted plot is O (n2) general memory utilization plot. Therefore, this proves the memory usage of the algorithm as order 2 polynomial.